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Adaptive Robust Mechanical System Control with Nonlinear Dynamic Friction Compensation

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Abstract

This study presents a dynamic friction compensation technique for a class of mechanical systems in the presence of dynamic friction eflects, based on the adaptive robust control (ARC) scheme. Both parametric uncertainties, such as unknown dynamic friction parameters and inertia load, and uncertain nonlinearities, such as external disturbances and unmeasurable state-dependent factors, may affect the mechanical system. The proposed ARC scheme uses both the structural information of the dynamic friction model and the available prior information on a physical system, such as the bounds of parameter variations and unmeasurable friction state, for high performance, in contrast to existing deterministic robust control (DRC) and robust adaptive control (AC) schemes. Specifically, certain projection-type alterations to the dynamic friction state observers are constructed by completely using the physical constraint on the unmeasurable dynamic friction state. This preserves the nominal estimate capabilities of the dynamic friction state observers while achieving a regulated dynamic friction state estimation procedure even in the presence of disturbances. The resultant controller achieves a guaranteed transient performance, meaning that the upper limit on the tracking error's absolute magnitude throughout the whole time history is provided and connected in a known way to certain controller design parameters. Additionally, asymptotic output tracking is realized when unknown nonlinearities are absent. The suggested scheme's excellent performance and reflectiveness are shown by comparative simulation results.

1.Introduction

It has been shown that friction is a dynamic nonlinear effect that negatively impacts the functionality of certain mechanical systems. The majority of friction's dynamic characteristics, including the Stribeck eflect, the friction lag, and the presliding displacement, to mention a few, take place in the so-called pres/iding and /o°-ve/ocity areas. Numerous academics have put out a variety of dynamic friction models to capture these effects. To describe the spring-like action during stiction, for instance, Dahl [1] presented a dynamic model. In order to represent friction eflects such the Stribeck eflect, hysteresis, presliding displacement, and variable break-away force, Canudas de Wit et al. [2] suggested a dynamic model. Additionally, a friction observer and friction correction for position and velocity monitoring have been built using this model. The goal of developing these very complex and precise models is to increase the accuracy of friction prediction, which will enable more effective friction compensation. However, since practically every physical system is susceptible to some degree of model uncertainty, it is often difficult to accurately reflect the nonlinear aspects of friction, regardless of how good these mathematical models may be. The causes of model uncertainties are typically divided into three groups: (i) repeatable or constant unknown quantities, such as the inertia load and unknown dynamic friction parameters; (ii) dynamic uncertainties resulting from the unmeasurable friction state; and (iii) non-repeatable unknown quantities, such as external disturbances and inaccurate modeling of specific physical terms. Deterministic robust control (DRC) [3, 4, 5, 6] and adaptive control (AC) [7, 8, 9] are two nonlinear control techniques that may be used to account for these uncertainties. In the presence of parametric uncertainties, dynamic uncertainties, and unknown nonlinearities, deterministic robust controllers may generally provide assured transient performance and ultimate tracking accuracy. However, the design is conservative and may involve either switching or infinite-gain feedback [5, 6] for asymptotic tracking because no effort is made to learn from past behavior to reduce the eflect of parametric and dynamic uncertainties; both approaches are unachievable in implementation, leading to a degraded performance [10]. Unlike DRC approaches, the adaptive controllers do not need infinite gain feedback to accomplish asymptotic tracking in

the absence of unpredictable nonlinearities. In particular, Canudas de Wit et al. demonstrated in [7] how the control structure suggested in [2] might be adjusted to accommodate certain unknown friction factors. A single parameter related to normal force variation was suggested for one adaptive controller, while a single parameter related to temperature variation was suggested for the second adaptive controller. In order to accommodate non-uniform fluctuations in the friction force, an adaptive controller using a nonlinear observerf'ter structure is presented in [8]. This controller assumes independent coefficient change as temperature changes. Similar results are obtained in [9], where several nonlinear reflects of the unmeasurable friction state are estimated using a duo-observer framework. Nevertheless, these adaptive controllers have two major shortcomings: they may not be resilient to shocks and their transient performance is unclear. During the first transitory phase, the system could respond slowly or have significant tracking mistakes. The closed-loop system may become unstable when disturbances are present [11].

This study uses the concept of adaptive robust control (ARC) to present a dynamic friction compensation technique for the position monitoring of a second order mechanical system [12, 13, 14]. The suggested approach effectively integrates the design techniques of DRC and AC by taking use of realistically feasible prior knowledge about physical systems, such as the boundaries of parameter fluctuations and the unmeasurable friction state. The resultant ARC controllers naturally overcome the practical limits of both DRC and AC while achieving the same goals. In particular, the widely used discontinuous projection mapping is used to modify the observer design proposed in [9] based on the available bounds on the unmeasurable friction state. This ensures that the friction state estimates always belong to a known bounded region regardless of whether the system is subject to disturbances and what kind of adaptation law is used. Therefore, for improved performance, the potentially destabilizing effect of friction state estimate mistakes may be addressed by some simple robust feedback. In the sense that the upper limit on the absolute value of the output tracking error across the whole time history is specified and connected to specific controller design parameters in a ùn form, the resultant controller achieves a guaranteed output tracking transient performance. Furthermore, despite potential parametric uncertainties in the system and dynamic friction model, asymptotic output tracking is obtained in the absence of unpredictable nonlinearities. The effectiveness and high performance of the suggested dynamic friction compensation technique will be shown via simulation results.

This is how the paper is structured. The problem formulation is presented in Section 2. The adaptive dynamic friction compensation design and the deterministic robust friction compensation design are presented in Sections 3 and 4, respectively, for comparison. The suggested ARC dynamic friction compensation technique is shown in Section 5. The comparative simulation findings are presented in Section 6, and the article is concluded in Section 7.

2 Problem Formulation

The mathematical model for a mechanical system in the presence of friction is assumed to be of the form

$$\frac{ds_1}{dí} = s_2, \quad (1)$$

$$m\frac{ds_2}{dí} = u — '' — A(s, z,, u, í), \quad (2)$$

where $s = [s_1\ s_2]^T$ represents the state vector consisting of the position and velocity, m denotes the unknown

mechanical inertia of the system, " is the friction, A represents the lumped unknown nonlinear functions such as disturbances and modeling errors, and u(í) is the control input. Using the dynamic friction model proposed by Canudas de Wit et o/. [2], the dynamic friction force '' is

$$'' = e_0 z - e_1 \frac{dz}{dí} - e_2 s_2, \qquad (3)$$

$$\frac{dz}{dí} = s_2 - \frac{¡s2¡}{p(s_2)} z, \quad (4)$$

where z represents the unmeasurable internal friction state, e_0, e_1, e_2 are unknown friction force pa-rameters that can be physically explained as the stiflness of bristles, damping coeÆcient associated with dz/dí, and viscous coeÆcient. The function $p(s_2)$ is assumed to be known and positive, and can be chosen to describe the Stribeck eflect [2]:

$$p(s) = \sigma_c - (\sigma_c - \sigma_{fi})e^{-(z\dot{z}=v_s)^2}, \quad (5)$$

where σ_C and σ_{fi} represent the levels of the normalized Coulomb friction and stiction force respectively, and r_s is the Stribeck velocity. This dynamic model considers the dynamic eflects of the friction as arising out of the deflection of bristles used to model the asperities between two contacting surfaces. The friction state essentially captures the average deflection of these bristles. Understandably, the state z(í) does not profler itself for direct measurement, which makes the dynamic friction compensation design very diÆcult, especially when some of the model parameters such as e_0, e_1, e_2 are not known in advance.

It is shown in [2] that the model has the following finite bristle deflection property:

P1 *If* $|z(0)| < \sigma_{fi}$, *then* $|z(í)| < \sigma_{fi}$, *ví* > 0.

Thus, in the following, it is assumed that $|z(í)| < \sigma_{fi}$, ví > 0. This property is physically intuitive and will be extensively used in the subsequent ARC controller designs for a guaranteed robust performance.

Substituting the friction dynamics (3) and (4) into (2), we rewrite the equation of motion as

$$\frac{ds_2}{dí} = \frac{1}{m}u - \frac{e_0}{m}z - \frac{e_1|s_2|}{m\,p(s_2)}z - \frac{e_1-e_2}{m^2}s - \frac{A}{m}, \quad (6)$$

In order to use parameter adaptation to reduce parametric uncertainties, it is necessary to linearly parameterize the equation in terms of a set of unknown parameters. For this purpose, we define an unknown parameter set $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T$, where $\theta_1 = 1/m$, $\theta_2 = e_0/m$, $\theta_3 = e_1/m$ and $\theta_4 = (e_1 - e_2)/m$. It should be noted that all these parameters are positive, which is an important fact utilized in the design of the controllers. Thus (6) can be rewritten as

$$\frac{ds_2}{dí} = \theta_1 u - \theta_2 z - \theta_3 \frac{|s_2|}{p(s_2)}z - \theta_4 s - \acute{A}, \quad (7)$$

where $\acute{A} = A/m$. For controller design, the following reasonable and practical assumptions are made:

Assumption 1 The eztent of porometric uncertointies is ùno«n, i.e.,

$$\theta \in \Omega_\theta = \{\theta : 0 < \theta_{nžn} < \theta < \theta_{noz}\},$$

«here $\theta_{nžn} = [\theta_{1nžn}, \ldots, \theta_{4nžn}]^T$ ond $\theta_{noz} = [\theta_{1noz}, \ldots, \theta_{4noz}]^T$ ore ùno«n.

Assumption 2 The uncertoin non/ineority A is bounded by o ùno«n function Æ(s, í) mu/tip/ied by on unùno«n but bounded time-vorying disturbonce d(í), i.e.,

$$A \in \Omega_A = \{A : |A(s, z, u, í)| < Æ(s, í)d(í)\}. \quad (9)$$

In the above and throughout the paper, the following notations will be used: •nžn for the minimum value of •, •noz for the maximum value of •, and the operation <2 for two vectors is performed in terms of the corresponding elements of the vectors. "• denotes the estimate of •.

Let s_d(í) be the desired motion trajectory, which is assumed to be known, bounded with bounded derivatives up to the second order. The objective is to synthesize a bounded control input u such that the actual position s_1 tracks s_d(í) as closely as possible in spite of various model uncertainties mentioned above.

To achieve the above objective, available tools for control of such a mechanical system - adaptive con-trol and deterministic robust control—will be applied first, from which one can gain insights about the development and the advantages of the proposed ARC.

3 Adaptive Control

In the absence of uncertain nonlinearities, i.e., A = 0, adaptive control [9] can be generalized to solve the problem. Define a switching-function-like quantity as

$e_2 = e"_1 — ke_1 = s_2 — s_{2eq}$, $s_{2eq} \equiv s"_d — ke_1$, (10)

where $e_1 = s_1 — s_d$ is the position tracking error, k is any positive feedback gain. If e_2 is small or converges to zero exponentially, then e_1 will be small or converge to zero exponentially since $G(s) = \frac{1}{eŽ(s)} \frac{e(s)}{s—k} = \frac{1}{s—k}$ is a stable transfer function. So the rest of the design is to make e_2 as small as possible. The structure of (7) motivates us to design the adaptive control law as follows:

$u = u_o — u_{s1}$, (11)

$$u = \frac{1}{8"_1}\left(—8" z" — 8" \frac{¡s2¡}{p(s2)} z" — 8" s — s" \right), \quad (12)$$

$u_{s1} = —k_s e_2$, (13)

where k_s is a positive design constant, $z"_0$ and $z"_1$ are estimates of the unmeasurable friction state z. After taking the time derivative of (10), substituting the expressions given by (7) and (11), and then simplifying the resulting expression, we obtain

$$\frac{de_2}{dí} = —8 k e — q^T 8' — 8 z' — 8 \frac{¡s2¡}{p(s2)} z' — Á, \quad (14)$$

where $q^T = [u_o, —z", \frac{|zŽ|}{p(z_1)} z", —s]$. $z'_0 = z"_0 — z$ and $z'_1 = z"_1 — z$ represent the estimation errors of the unmeasurable friction state. $8" = 8" — 8$ represents the parameter estimation error due to parametric uncertainties. When A = 0, the eflect of dynamic uncertainties can be canceled by using the following observers:

$$\frac{dz"_0}{dí} = s — \frac{¡s2¡}{p(s_2)} z" — q e, \quad (15)$$

$$\frac{dz'_1}{dt} = s - \frac{|s_2|}{2}z'' - q\frac{|s_2|}{p(s_2)^{1}}e, \quad (16)$$

where q_0 and q_0 are two positive design constants. The eflect of parametric uncertainties, on the other hand, can be eliminated by learning from the past information through the following parameter adaptation mechanisms

$$\hat{\theta} = F\varphi e_2, \quad (17)$$

where $F > 0$ is the adaptation rate matrix.

The closed-loop stability of the adaptive controllerJobserver can be proved via a Lyapunov-type argument. Specifically, define the following nonnegative function

$$V_o = \frac{1}{2}e_2^2 + \frac{1}{2q_2}z_0^2 + \frac{1}{2q_3}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \quad (18)$$

By using similar techniques as in [9], it can be proved that all the signals are bounded and asymptotic position tracking is achieved.

Remark 1 In f9], the inertio m is ossumed to be ùno«n. In oddition, the t«o observer design constonts q_0 ond q_1 ore set to be 1, «hich con be vie«ed os o specio/ cose of the obove design.

4 Deterministic Robust Control

Adaptive control has two main drawbacks - unknown transient performance and possible non-robustness to disturbances. The system may have large tracking errors during the initial transient period or have a sluggish response. In the presence of uncertain nonlinearity A, the closed-loop system may be unstable [11] , since parameter adaptation may learn in a wrong way or the observer states may go unbounded. In contrast, instead of using parameter adaptation and friction observers, deterministic robust control (DRC) only uses robust feedback to attenuate the eflect of various model uncertainties for a guaranteed performance as done in the following. Consider the following DRC law

$$u = u_o - u_{s1} - u_{s2}, \quad (19)$$

where u_o is given by (12) but with fixed state estimate $\hat{z}'_0 = \hat{z}'_1 = \hat{z}'(0)$ and parameter estimate $\hat{\theta}'' = \hat{\theta}''(0)$ (i.e., a fixed feedforward friction compensation), u_{s1} is given in (13) and u_{s2} is a robust control term to be synthesized later. After substituting the control law into (7), we obtain the following error dynamics

$$\frac{de_2}{dt} = -\theta_1 k_s e_2 - q^T\tilde{\theta}' - \theta_0 z' - \theta_0 \frac{|s_2|}{p(s_2)^{3}}z' - A - \theta_0 u_0 + \theta_0 u_{s2}, \quad (20)$$

where $q^T = [u_0, -z'(0), \frac{|z\check{Z}|}{p(z_2)} z'(0), -s]$, $z' = z'(0) - z$ and $8' = 8'(0) - 8$. The essence of DRC is that the added robust control term u_{s2} can be synthesized to dominate the model uncertainties coming from parametric uncertainties, dynamic uncertainties and uncertain nonlinearities and to attenuate their eflects. Specifically, noting Assumption 1, Assumption 2 and Property P1, a robust control function u_{s2} can be synthesized such that the following condition is satisfied:

$$e_2[-q^T 8' - 8_0 z' - 8_0 \frac{|s_2|}{p(s_2)} z' - A - 8_1 u] < c_0 - c_1 d^2, \quad (21)$$

where c_0 and c_1 are two design parameters which may be arbitrarily small. By doing so, the derivative of the following positive semi-definite function

$$V_s = \frac{1}{2} e_2^2. \quad (22)$$

is

$$\dot{V}_s = -8_1 k_s e_2^2 - e_2[-q^T 8' - 8_0 z' - 8_0 \frac{|s_2|}{p(s_2)} z' - A - 8_1 u] < -\check{z}_V V_s + c_0 + c_1 d^2, \quad (23)$$

where $\check{z}_V = 2k_s 8_{1nzn}$. (23) implies that

$$V_s(t) < \exp(-\check{z}_V t) V_s(0) + \frac{c_0 + c_1 \|d\|_\infty^2}{\check{z}_V}[1 - \exp(-\check{z}_V t)], \quad (24)$$

where $\|d\|_\infty$ stands for the infinity norm of $d(t)$. Thus $V_s(t)$ exponentially decays and is ultimately bounded. The exponentially converging rate $\check{z}_V$ and the final tracking accuracy index ($|V_s(\infty)| < e_0 + e_1 |d|_\infty^2$) can be freely adjusted by the design parameters c_0, c_1 and k_s in a known form. In other words, transient performance and final tracking accuracy are guaranteed.

Remark 2 One ezomp/e of o smooth u_{s2} sotisfying (21) con be found in the fo//o«ing «oy. Łet h_0, h_1 ond h_2 be ony smooth bounding functions sotisfying the fo//o«ing conditions,

$$h_0 > \|q_0\| \|8_{noz} - 8_{nzn}\|, \quad (25)$$

$$h_1 > 8_{2noz}(z_{noz} - z_{nzn}), \quad (26)$$

$$h_2 > 8_{3noz} \frac{|s_2|}{p(s_2)}(z_{noz} - z_{nzn}). \quad (27)$$

«here $z_{noz} = \theta_{fi}$ ond $z_{nzn} = -\theta_{fi}$ in vie«ing Property P1. Then, u_{s2} con be chosen os

$$u_{s2} = -\frac{1}{48_{1nzn} c_0^0 c_0^1 c_0^2 c_1^2}\left(\frac{3}{c_0^0} h_2^2 - \frac{3}{c_0^1} h_2^2 - \frac{3}{c_0^2} h_2^2 - \frac{1}{c_1^2} Æ^2\right) e_2. \quad (28)$$

Using the some technique os in f1S], «e con sho« thot

$$e_2[-q^T 8' - 8_0 z' - 8_0 \frac{|s_2|}{p(s_2)} z' - A - 8_1 u]$$

$< |e_2|h_0 - |e_2|h_1 - |e_2|h_2 - |e_2|Æd - e_2 8_1 u_{s2}$

$< -(\frac{h_0}{2g_{c0}} 3 |e| - \frac{y_{c0}}{2})^2 - (\frac{h_1}{2g_{c0}} 3 |e| - \frac{y_{c0}}{2})^2 - c_0 \frac{g}{2}$

$-(\frac{1}{2g_{c1}} Æ|e| - g_c d)^2 - c_1 d^2$

$< c_0 - c_1 d .$ (29)

Thus (21) is sotisfied. Other smooth ezomp/es con be «orùed out in the some «oy os in f1S, 1J].

Adaptive Robust Control

Since both transient error and final tracking error may potentially be made as little as possible by raising ks and reducing c0 and c1, it seems that the DRC described above has addressed the robust control challenge. Nevertheless, this will lead to a rise in feedback gains, which in turn will enhance the closed-loop system's bandwidth. Every physical system has a limited bandwidth because of overlooked elements like high-frequency dynamics and sampling rate. As a result, DRC's practical precision is constrained. Dynamic friction state observers and parameter modification, similar to adaptive control, are required to further enhance performance by lowering model uncertainty. It seems that the parameter adaptation rule (17) and state observers (15) and (16) may be used to update the parameter and friction state estimates in (19). Unfortunately, there are significant conflicts between the two design approaches, thus this straightforward combination is ineffective. Specifically, in the presence of unknown nonlinearity A, the estimations of the friction state and parameters in adaptive control in section 3 may become unbounded. Therefore, eliminating the potential destabilizing effect of parameter adaptation and state estimation inaccuracy while maintaining their nominal learning and estimating capabilities is one of the main challenges in merging the adaptive control and DRC systems. It has been shown in [12] that this objective may be accomplished for a class of nonlinear systems transformable to the semi-strict feedback form using the commonly utilized discontinuous projection mapping for robust parameter adaptation. Nevertheless, [12] only takes into account the eflect of constant unknown parameters; dynamic uncertainties, such as the unmeasurable friction state, are not taken into account. For a class of second order mechanical systems where the boundaries of parameter fluctuations and internal friction state are known as described in Section 2, we will expand the discontinuous projection based ARC design [12] and suggest a dynamic friction compensation strategy.

5.1 Discontinuous Projection Mapping

By exploiting practically reasonable prior information on the physical systems such as the bounds of pa-rameter variations and internal friction state, a discontinuous projection based ARC design is constructed to solve the robust tracking control problem. Specifically, the parameter estimate $8''$ is updated through a parameter adaptation law having the form given by

$$8 = \text{Proj}_8(Fqe_2),$$

(30)

where $Proj_8(Fqe_2) = [Proj_8((Fqe_2)_1), \ldots, Proj_8((Fqe_2)_4)]^T$, and $(Fqe_2)_i$ represents the i-th component of Fqe_2. The unmeasurable friction state z is estimated by the following robust observers with projection type modifications respectively

$$\frac{dz'0}{dt} = Proj\ (s - \frac{|s2|}{p(s_2)} z'' - q\ e\ j, \quad (31)$$

$$\frac{dz_1}{dt} = Proj\ (s - \frac{|s_2|}{p(s_2)} z'' - q \frac{|s2|}{p(s_2)} e\ j. \quad (32)$$

In (30) to (32), the projection mapping $Proj_{Ö}(\bullet)$ is defined by [15, 16]

$$Proj_{Ö}(\bullet) = \begin{cases} 0 & \text{if } Ö = Ö_{noz} \text{ and } \bullet > 0 \\ 0 & \text{if } Ö = Ö_{nžn} \text{ and } \bullet < 0 \\ \bullet & \text{otherwise} \end{cases} \quad (33)$$

where Ö is a symbol that can be replaced by 8_1 - 8_4, z_0 or z_1, and $z_{0noz} = z_{1noz} = "_{fi}$ and $z_{0nžn} = z_{1nžn} = —"_{fi}$ in viewing Property P1. As an example, the detailed expression for (31) is given by

$$d\dot{z}_0 = \begin{cases} 0 & \text{if } z"_0 = z_{0noz} \text{ and } s_2 — p(zŽ)\, z"_0 — q_0 e_2 > 0 \\ — & \text{if } z'_0 = z_{0nžn} \text{ and } s_2 \frac{|zŽ|}{p(zŽ)} \beta — q_0 e_2 < 0 \\ dí & \text{otherwise} \end{cases} \quad (34)$$

Noting Property P1 and using similar arguments as in [15, 16, 10], we can show that the above projection mappings have the following nice properties

P2 $Ö \in G_Ö = \{Ö : Ö_{nžn} < Ö < Ö_{noz}\}$ (35)

P3 $\beta^T (F^{-1} Proj_\beta(Fqe_2) — qe_2) < 0$ (36)

P4 $z^T (Proj_{\dot{z}_0} (s_2 — \frac{|s_2|}{p(s_2)} z" — q_0 e_2)) — (s_2 — \frac{|s_2|}{p(s_2)} z" — q_0 e_2)\} < 0$ (37)

P5 $z^T (Proj_{\dot{z}_1} (s_2 — \frac{|s_2|}{p(s_2)} z" — q_1 \frac{|s_2|}{p(s_2)} e_2)) — (s_2 — \frac{|s_2|}{p(s_2)} z" — q_1 \frac{|s_2|}{p(s_2)} e_2)\} < 0$ (38)

5.2 ARC Controller Design

The proposed ARC law has a similar structure as the DRC law (19)

$$u = u_o — u_{s1} — u_{s2}, \quad (39)$$

where u_o is given by (12) but with β updated by (30), and $\dot{z}_0$ and $\dot{z}_1$ updated by (31) and (32) respectively. u_{s1} is given by (13), and u_{s2} is synthesized in a similar way as in DRC design as follows. Substituting the ARC control law into (7), we obtain the following error dynamics

$$\frac{de_2}{dt} = -\beta_1 k_s e_2 - q^T\beta_2 z_2 - \beta_2 \frac{\|s_2\|}{p(s_2)} z_2 - A_0 \beta_3 u_1 + \beta_1 u_{s2} \quad (40)$$

As in (21), u_{s2} is synthesized to satisfy the following constraint,

$$e_2^T[-q^T\beta_2 z_2 - \beta_2 \frac{\|s_2\|}{p(s_2)} z_2 - A_0 \beta_3 u_1 + \beta_1 u_{s2}] < c_2 \|e_2\| + d_2 \quad (41)$$

Such a robust control term u_{s2} always exists since both parameter and state estimates belong to some known bounded regions by Property P2. To ensure that the robust control term u_{s2} does not interfere

with the nominal estimation process of parameter adaptation and friction state observer, the following passivity-like requirement is also imposed:

ii $e_2 u_{s2} < 0$. (42)

Example of u_{s2} satisfying (41) and (42) is given by (28) with the bounding function h_0 redefined as $h_0 > kqkk8_{noz} — 8_{nžnk}$.

Theorem 1 Vith the ARC contro/ /o« (S9), porometer odoptotion /o« (SO), ond friction stote observers (S1) ond (S2), the fo//o«ing resu/ts ho/d.-

A. In genero/, o// signo/s ore bounded. Furthermore, V_s is bounded obove by

$$c_0 — c_1 kdk^2$$

$$V_s(í) < esp(—žvíí) V_s(0) — \frac{}{žv} [1 — esp(—žvíí)]. \quad (43)$$

B. If ofter o finite time $í_f$, A = 0, i.e., in the presence of porometric uncertointies ond dynomic friction on/y, then, in oddition to resu/ts in A, osymptotic position trocùing (or zero fino/ trocùing error) is o/so ochieved. a

Proof.- See Appendix.

Remark 3 Resu/ts in A of Theorem 1 indicote thot the proposed contro//er hos on ezponentio//y converging tronsient performonce «ith the ezponentio//y converging rote $ž_V$ ond the fino/ trocùing error indez (i.e., $(¡V(>)_¡ < e0 — e1 \frac{|d|}{Ž})$ being ob/e to be odjusted vio certoin contro//er porometers c , c ond k free/y

s *V* *0* *1* *s*

in o known form. Theoretico//y, this resu/t is «hot o «e//-designed robust contro//er con ochieve. In foct, «hen the porometer odoptotion /o« (SO) ond stote observers (S1) (S2) ore s«itched off, the proposed ARC /o« becomes o deterministic robust contro/ /o« ond Resu/ts A of the Theorem remoin vo/id f1O, 1J].

B of Theorem 1 imp/ies thot «ithout using infinite feedbocù goin, the contro//er moy hove o very smo// trocùing error due to the reduced porometric uncertointies ond dynomics uncertointies. Theoretico//y, Resu/t B is «hot o «e//-designed odoptive contro//er con ochieve. ö

Remark 4 It is seen from (JS) thot tronsient trocùing error is offected by the initio/ vo/ue $V_s(0)$. To further reduce tronsient trocùing error, the ideo of trojectory initio/izotion f1O] con be used to render $V_s(0) = 0$. ö

Remark 5 In the obove, the effect of three unùno«n dynomic friction mode/ porometers, e_0, e_1 ond e_2, is considered, «hich essentio//y coptures the situotion thot voriotion of friction is coused by temperoture chonges ond moterio/ «eorf7]. It is noted thot the obove ARC design procedure con be stroightfor«ord/y genero/ized to the situotions «here voriotion of friction is coused by the chonge of normo/ force f7]. The detoi/s ore omitted. ö

6 Comparative Simulation Results

In order to illustrate the proposed ARC algorithm and compare it with the previous friction compensation designs, comparative simulation results are obtained for the model described by (1)-(5), in which the numerical values of the parameters are taken from reference [8]
$m = 0.125$kp-m^2 $e_0 = 12$Nm/rad $e_1 = 0.1$Nm-s/rad
$e_2 = 13.2$Nm-s/rad $''_c = 3.24$ $''_s = 8.45$ $r_s = 3.0$.
For comparison purpose, the mechanical inertia m is assumed to be known as in [9] . Thus there are only three parameters to be adapted and we redefine the unknown parameter set as $8 = [e_0, e_1, \&]^T$, where $\& \equiv e_1 — e_2$. The bounds describing the uncertain ranges are given by $8_{nžn} = [0, 0, 0]^T$, $8_{noz} = [20, 0.3, 2]^T$, $z_{nžn} = —8.45$, $z_{noz} = 8.45$ and $Æ = 1$. Applying the adaptive robust design procedure in section 5, we have the following control law:

$$q_0 = [—z'' \quad \frac{¡s2¡}{2} z'' — s\,] \,, \quad (44)$$

$$p(s_2)^{-1}$$

$$h_0 > kqk8_{noz} — 8_{nžn}k, \quad (45)$$

$$h_1 > 82noz(znoz — znžn), \quad (46)$$

$$h_2 > 3noz\,8\,\frac{¡s2¡}{p(s2)}(z - z). \quad (47)$$

$$noz \quad nžn$$

$$u_o = —q^T8'' — ms''_{2eq}, \quad (48)$$

$$u_{s1} = —k_s e_2, \quad (49)$$

$$u_{s2} = —\frac{1}{4\,c0}(\frac{3}{0}h^2 — \frac{3}{c0}h^2 — \frac{3}{c_0^2}h^2 — \frac{1}{c_1^2}Æ^2)\bar{e}\,, \quad (50)$$

$$1$$

$$u = u_o — u_{s1} — u_{s2}. \quad (51)$$

In the simulation, three diflerent controllers synthesized early are compared:

ARC: the proposed adaptive robust controller given by (44)-(51) and (30)-(32).
AC: the adaptive controller proposed in [9] , which is given by (51), (15)-(17), but with $u_{s2} = 0$. For comparison purpose, initial conditions and common design parameters of ARC and AC are kept the same as follows:

$$z(0) = ż_0(0) = ż_1(0) = s_1(0) = s_2(0) = 0,\ 8'(0) = [6, 0.05, 0.71]^T,$$
$$k = 5,\ k_s = 5,\ F = diap[2000, 100, 50],\ ą_0 = ą_1 = 1,\ c_1 = c_2 = 4 « 10^3. \quad (52)$$

DRC: the deterministic robust controller - the same control law as that in ARC but without using param-eter adaptation and state observers. Initial conditions and design parameters remain unchanged except $F = 0$ and $k = k_s = 25$.

The desired position trajectory (see Figure 1) is selected to be [8]

$s_d = 1.0 \tan^{-1}(4 \sin(0.5í)(1 - \exp(-0.01í^3))$ rads. (53)

The following two typical cases are considered:

Case 1: No Disturbance

To test the nominal performance of all three controllers, simulation is first run in the absence of uncertain nonlinearities, i.e., A = 0 is used in the simulation. The tracking errors of all three controllers are shown in Figure 2. It is seen that the proposed ARC scheme has the advantages of both DRC and AC: the transient period of ARC is comparable to that of DRC while asymptotic tracking is achieved as in AC, which agrees with the theoretical results obtained in Theorem 1. Overall, AC has the worst transient performance and DRC has a large non-converging final tracking error. The control inputs of AC and ARC are given in Figure 3. It shows that due to the added robust term u_{s2}, ARC delivers a stronger control action during transient period in order to guarantee transient performance. The control eflort of DRC is not shown in the figure. It is about 15 times as large as other two because of large tracking error and control gains. Figure 4 and Figure 5 show that ARC has a more stable parameter estimation process and state estimation process than that of AC.

Case 2: With Disturbance

To test the performance robustness of the controllers to uncertain nonlinearities, a random disturbance signal with an amplitude of 6.25(Nm) is added to the system at í = 0 sec and remove at í = 20 sec. The tracking errors of AC and ARC are shown in Figure 6. It is seen that ARC achieves a much smaller tracking error during the period when disturbance occurs. In addition, ARC recovers to its nominal performance much faster after the disturbance is removed at í = 20 sec. One reason is that the disturbance is dealt with via the robust feedback term u_{s2}. The other reason is that, due to the use of projection mappings, ARC has a more stable parameter estimation process and state estimation process, which are shown in Figure 7 and Figure 8. All these results well illustrate the eflectiveness of the proposed ARC control strategy in handling dynamic friction force and disturbances.

7.**Conclusion**

This work presents an adaptive robust control (ARC) technique based on discontinuous projection for a class of mechanical systems in the presence of dynamic friction eflects. The suggested ARC system, like other current adaptive dynamic friction compensation (AC) schemes, builds nonlinear observers to recover the unmeasurable friction state using the structural information of the friction dynamics. This eliminates the effect of dynamic uncertainties in the absence of uncertain nonlinearities and improves performance; asymptotic output tracking is accomplished without using discontinuous or infinite gain feedback control laws in the presence of parametric uncertainties and unmeasurable dynamic friction state. Additionally, the proposed scheme employs projection mappings for a stable parameter and friction state estimation process in the presence of uncertain nonlinearities by utilizing available prior information on the system, such as the bounds of parameter variations and unmeasurable friction state. It also uses certain robust feedback to attenuate the reflect of both estimation errors and uncertain nonlinearities for a guaranteed robust performance in general. Because of this, the proposed solution naturally overcomes the practical limits of both quick robust feedback

and adaptive dynamic friction correction. The eflectiveness of the suggested strategy is shown via comparative simulation results.

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